LA-UR- 04 - 0419

Approved for public release;
distribution is unlimited.

Title: Bayesian Model Selection for a Finite Element Model of a Large Civil Aircraft

Author(s): Francois M. Hemez
Amanda C. Rutherford

Submitted to: International Modal Analysis Conference 2004

Los Alamos
NATIONAL LABORATORY

Bayesian Model Selection for a Finite Element Model of a Large Civil Aircraft

IMAC04 Presentation Abstract

F.M. Hemez, A.L. Cundy

Nine aircraft stiffness parameters have been varied and used as inputs to a finite element model of an aircraft to generate natural frequency and deflection features (Goge, 2003). This data set (147 input parameter configurations and associated outputs) is now used to generate a metamodel, or a fast running surrogate model, using Bayesian model selection methods. Once a forward relationship is defined, the metamodel may be used in an inverse sense. That is, knowing the measured output frequencies and deflections, what were the input stiffness parameters that caused them?

UNCLASSIFIED

Bayesian Model Selection for a Finite Element Model of a Large Civil Aircraft

A.C. Rutherford and F.M. Hemez
Los Alamos National Laboratory, ESA-WR
LA-UR 04-xxxx

IMAC-XXII, January 26-29, 2004 — Dearborn, MI

UNCLASSIFIED

Los Alamos NATIONAL LABORATORY

ENGINEERING SCIENCES & APPLICATIONS — WEAPON RESPONSE

UNCLASSIFIED

Acknowledgements

- D. Göge, German Aerospace Center (DLR), who provided the extensive dataset that was used to complete this work.

- Dr. D. Bingham, Simon Fraser University, who provided guidance on the use of the Gibbs sampler.

UNCLASSIFIED

Los Alamos NATIONAL LABORATORY

ENGINEERING SCIENCES & APPLICATIONS — WEAPON RESPONSE

UNCLASSIFIED

Motivation

- Explore the use of parameter screening and metamodel design through a Bayesian model selection framework.
- Demonstrate usefulness of these approaches for uncertainty propagation and model updating (computationally intensive techniques, used often in the aerospace & automotive communities).
- Potentially useful in terms of design; parameters leading to desirable flutter characteristics can be identified.

UNCLASSIFIED

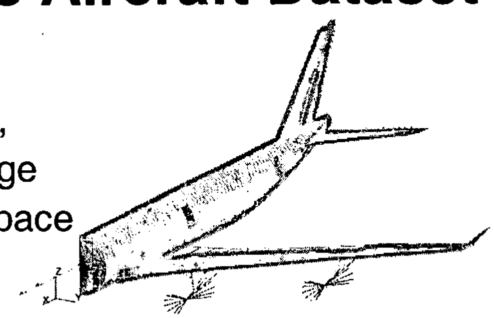
ENGINEERING SCIENCES & APPLICATIONS — WEAPON RESPONSE

Los Alamos NATIONAL LABORATORY

UNCLASSIFIED

Overview of the Aircraft Dataset

- Finite element model of a large civil aircraft, constructed by D. Göge of the German Aerospace Center (DLR).



- Using this model, a face-centered cubic design (size 147) was generated relating nine stiffness parameters to the first natural frequency and six associated modal displacements.

UNCLASSIFIED

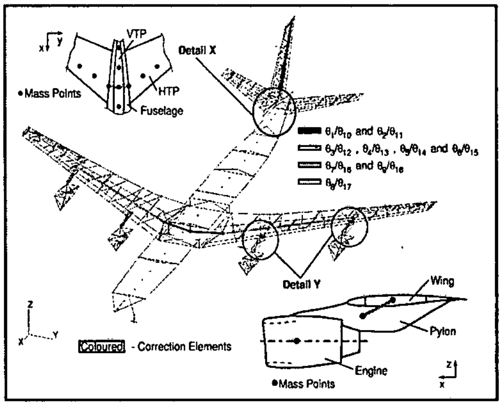
ENGINEERING SCIENCES & APPLICATIONS — WEAPON RESPONSE

Los Alamos NATIONAL LABORATORY

UNCLASSIFIED

The Stiffness Parameters

Parameter	Location	Type
$X_1=X_{10}$	Fuselage/wing connection (I/r)	I_{min}
$X_2=X_{11}$	Fuselage/wing connection (I/r)	I_{max}
$X_3=X_{12}$	Wing/pylon connection at outer engine (I/r)	I_{min}
$X_4=X_{13}$	Wing/pylon connection at outer engine (I/r)	I_{max}
$X_5=X_{14}$	Wing/pylon connection at inner engine (I/r)	I_{min}
$X_6=X_{15}$	Wing/pylon connection at inner engine (I/r)	I_{max}
$X_7=X_{16}$	Fuselage/HTP connection (I/r)	I_{min}
$X_8=X_{17}$	Wing (I/r)	E
$X_9=X_{18}$	Fuselage/HTP connection (I/r)	I_{max}

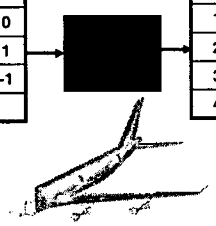


Note that parameters are symmetric about the length of the aircraft, so only include 9 of them.

UNCLASSIFIED

More on the Aircraft Dataset

Run	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
1	0	0	0	0	0	0	0	0	0
2	1	-1	-1	1	1	1	1	-1	1
3	1	1	1	1	-1	1	1	1	-1
4				Etc...					



Run	F1	M1	M2	M3	M4	M5	M6
1	.15	-.37	-.15	.32	-.08	.05	.18
2	.45	-.79	.89	-.21	-.77	.88	.75
3	.93	-.84	.83	-.42	-.16	.61	.85
4				Etc...			

Each stiffness input parameter set at one of three normalized values, 147 combinations in total (according to a face-centered cubic design).

Finite element model yields the seven output features (normalized between -1 and 1) for each of the 147 runs.

Overview of Model Selection

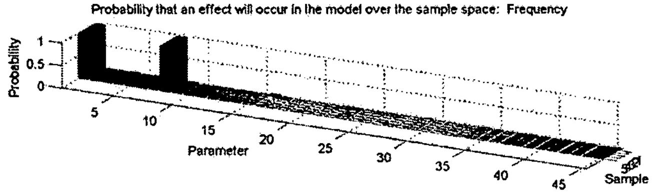
- A polynomial metamodel is generated for each of the available input parameter-output feature pairs using a Bayesian model selection algorithm.
- Error of the metamodel is assessed.
- The model is then used in an inverse sense with test data for identification of stiffness parameters.

Use of a metamodel is faster than running a finite element model (solves one equation as opposed to many), and hence it may be sampled more extensively.

Advantage of Bayesian Model Selection

- We are able to fit a model having a polynomial form (main effects and interactions, in our case **<u>45 terms</u>**) and assess how likely each term is to be in the model.

$$Y = \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_{10} X_1 X_2 + \ldots$$



- We can also assess these models multiple times since the calculation time is small.

How to Use the Dataset?

147 stiffness parameter and output feature pairings

Randomly sample from 112 a set of 92 to *fit* the model.

Evaluate the model with the 20 that are left.

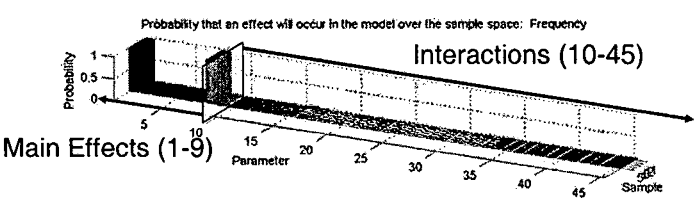
Do this "many" (10,000) times and compile statistics about the resulting models.

Check how each model does with the evaluation set.

Reserve the 35 for model *validation*

For comparison of individual models, use the validation set of 35, which is always the same.

Effect Probabilities

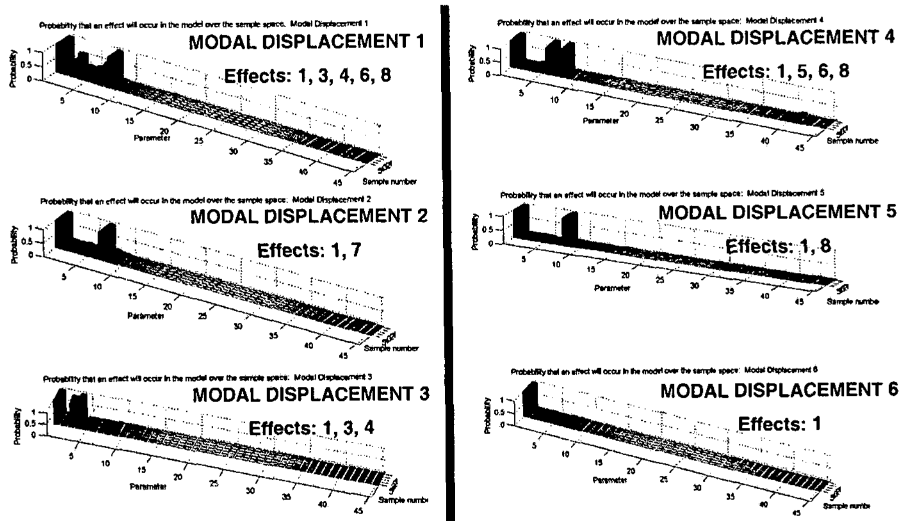


$$F_1 = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + \text{interactions}$$

- Most probable effects for FREQUENCY are effects 1 and 8, which are the main effects representing the ***fuselage to wing connection*** and the ***wing stiffness***.
- For all features, only main effects were probable. Interactions had very low probability.

UNCLASSIFIED

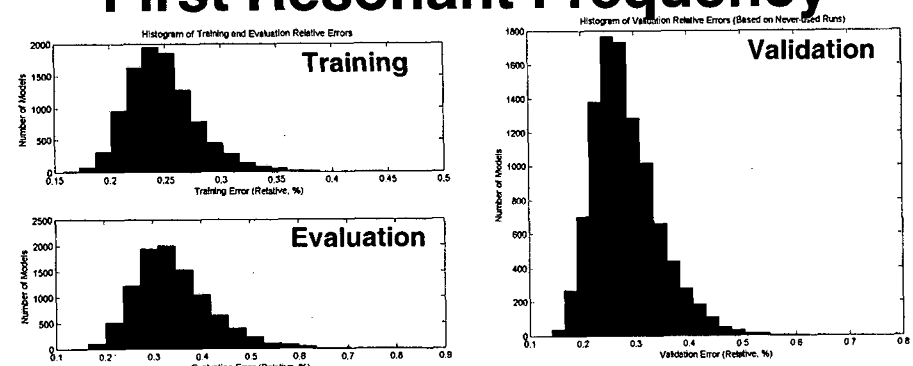
Effect Probabilities



UNCLASSIFIED

UNCLASSIFIED

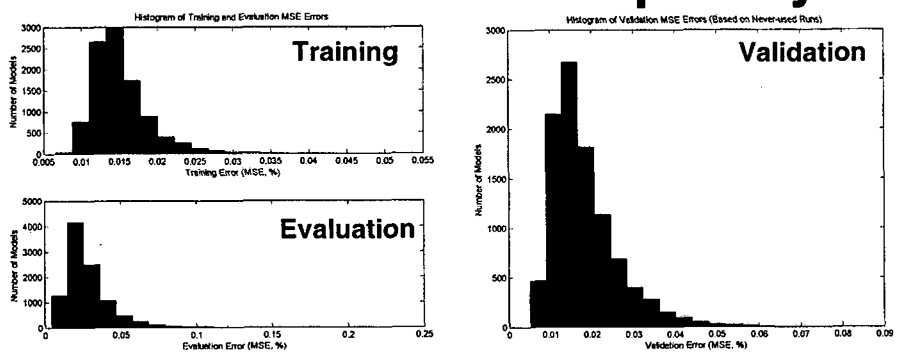
Mean Relative Errors for the First Resonant Frequency



The maximum mean relative error for any of the sets (Training, Evaluation and Validation) is *less than 1%*.

UNCLASSIFIED

Mean Square Error for the First Resonant Frequency



The maximum mean square error for any of the sets (Training, Evaluation, Validation) is generally *<u>less than 0.1%</u>*.

Use of Metamodels for Stiffness Parameter Calibration

- Can think of this in a design sense:
  - How to change stiffness parameters for a desired change in frequency.
- Can also think of this in a model updating sense:
  - How to change stiffness parameters to better match measured frequencies and mode shapes.

In this demonstration, we limit ourselves to identifying stiffness parameters for an experimentally measured first frequency.

UNCLASSIFIED

Inverse Problem Formulation

- We have 10,000 metamodel formulations relating the 9 stiffness parameters to the first frequency:

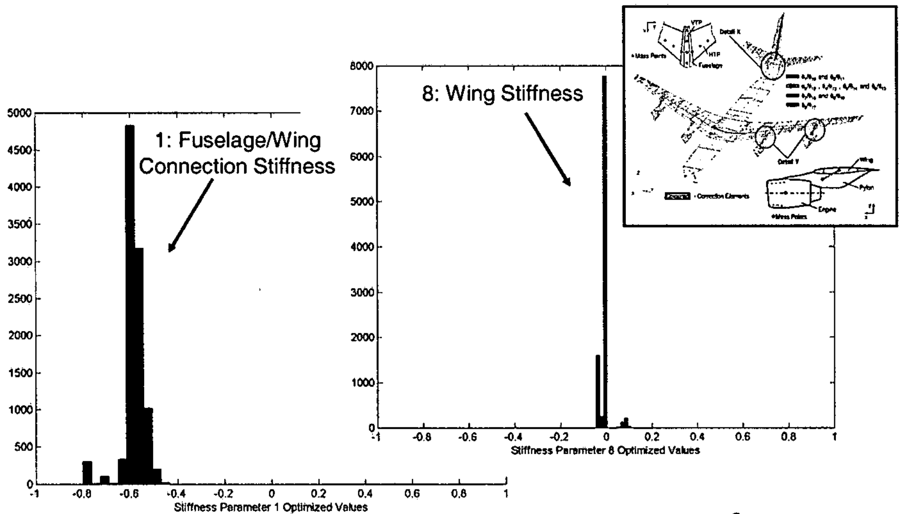
$$\begin{bmatrix} \omega^{(1)} = \beta_1^{(1)} x_1 + \beta_2^{(1)} x_2 + \ldots \\ \vdots \\ \omega^{(10,000)} = \beta_1^{(10,000)} x_1 + \beta_2^{(10,000)} x_2 + \ldots \end{bmatrix} \longrightarrow \varepsilon_i = \left(\omega_{measured} - \omega^{(i)}\right)^2$$

- We work to minimize a squared error cost function to determine what the stiffness parameters are for each of the 10,000 models.
- We then compile statistics on stiffness parameter values (solutions are not unique).

Stiffness Distributions

- Note that all parameters were initialized at their nominal normalized value (0).
- Parameters were all adjusted within the expected range of variation (between the normalized values of -1 and +1).
- For brevity, we examine parameters 1 and 8 (fuselage/wing connection and wing stiffness) because they were shown to be important to the first resonant frequency.

Stiffness Distribution Results



Calibration Under Uncertainty

- Measurement errors are propagated through parametric calibration to assess the effect of experimental uncertainty:

$$\begin{bmatrix} \omega^{(1)} = \beta_1^{(1)} x_1 + \beta_2^{(1)} x_2 + \ldots \\ \vdots \\ \omega^{(10,000)} = \beta_1^{(10,000)} x_1 + \beta_2^{(10,000)} x_2 + \ldots \end{bmatrix} \longrightarrow \varepsilon_i = \left(\frac{\omega_{measured} - \omega^{(i)}}{\sigma_{Test}} \right)^2$$

where $\omega_{measured}$ is now sampled from a Gaussian distribution $N(\mu_{Test}; \sigma_{Test})$ with $\sigma_{Test} / \mu_{Test} = 1\%$.

- As before, we then compile statistics on stiffness parameter values (solutions are not unique).

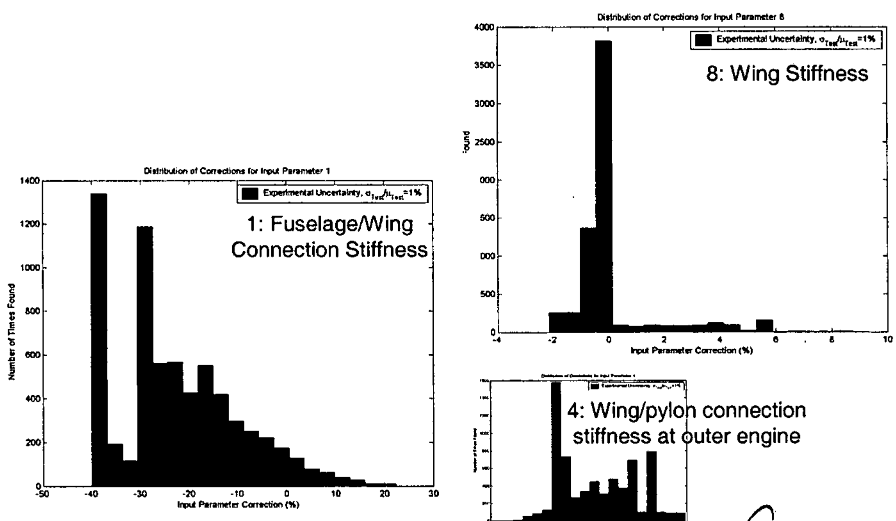
Stiffness Distributions

- Experimental uncertainty has a significant effect on the calibration results:

	Calibration With no Experimental Uncertainty		Calibration With 1% Experimental Uncertainty	
Parameter	Mean Change	Variance	Mean Change	Variance
1	-29.47%	7.8%	-22.69%	56.2%
2	3.00%	106.3%	2.29%	143.3%
3	3.11%	118.7%	2.30%	151.7%
4	9.44%	55.3%	6.37%	93.5%
5	2.95%	121.8%	2.36%	148.1%
6	2.92%	137.7%	2.45%	157.1%
7	2.89%	134.1%	2.22%	153.2%
8	-0.41%	245.7%	0.11%	1,401.0%
9	2.86%	135.6%	2.59%	163.6%

The correlation between parameters 1 & 4 increases with experimental uncertainty.

Stiffness Distribution Results

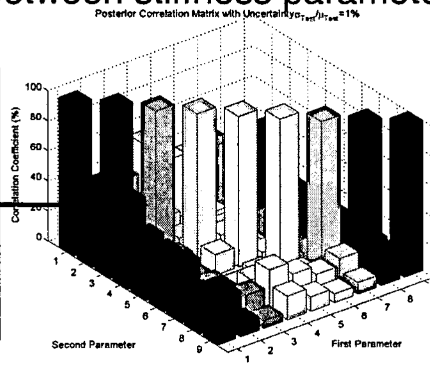


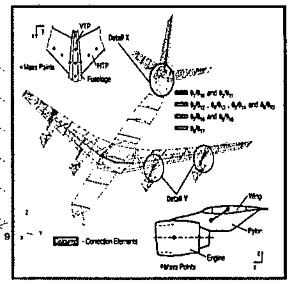
Posterior Correlation

- No significant posterior correlation is detected as the level of experimental uncertainty increases, except between stiffness parameters 1 & 4:

Parameter 1:
Fuselage/wing connection.

Parameter 4:
Wing/pylon connection at outer engine.





Multi-Objective Optimization

- The data available to us was the measured first resonant frequency and 6 associated modal displacements.
- Multi-parameter optimization did not work well – a check of the correlation coefficient matrix reveals why – all features are highly correlated, and not providing linearly independent information.
- Need to have further frequency data (that is presumably less correlated) for multi-parameter optimization to work.

	F1	M1	M2	M3	M4	M5	M6
F1	1.0	-0.9	0.8	-0.8	-0.6	0.9	.09
M1		1.0	-0.9	0.7	0.8	-0.9	-0.9
M2			1.0	-0.8	-0.8	0.9	.09
M3				1.0	0.6	-0.8	-0.8
M4		SYM.			1.0	-0.9	-0.8
M5						1.0	0.9
M6							1.00

UNCLASSIFIED

Conclusions

- Bayesian model selection provides probabilistic data about stiffness parameters and how important they are to various output features.
- Metamodel format means that the model can be stochastically analyzed very quickly.
- For all output features, there was very little error in the forward model sense.
- Inverse problem formulation yielded a distribution of input parameters that were within the expected range of variation. This information could be used in a model updating sense or in a design sense.
- Multi-parameter optimization could be utilized in future analyses, provided output features are not highly correlated.

UNCLASSIFIED

ENGINEERING SCIENCES & APPLICATIONS — WEAPON RESPONSE

Los Alamos NATIONAL LABORATORY

UNCLASSIFIED

Future Work

- Obtain higher resonant frequencies and mode shapes from the FEA model for the 147 combinations of stiffness parameters.
- Couple metamodels with a flutter analysis and utilize stiffness parameter identification capability for flutter design purposes.

UNCLASSIFIED

ENGINEERING SCIENCES & APPLICATIONS — WEAPON RESPONSE

Los Alamos NATIONAL LABORATORY